Comment on N/D equations and the ρ resonance

E. P. Tryon
Department of Physics and Astronomy, Hunter College of The City University of New York, New York, New York 10021
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The criticism by Dilley of my earlier argument against the N/D generation of a ρ resonance is acknowledged to be well founded. It is shown, however, that the gap-matching method favored by Dilley is strongly biased in favor of generating a ρ. Hence the N/D calculations by myself and by Dilley provide little if any evidence concerning the dynamical origin of the ρ.

In the preceding paper,[1] Dilley has criticized a calculation[2] which I previously offered as evidence that the ρ resonance is not generated by forces in the $\pi\pi$ channel. Dilley's criticism is well founded. In particular, solutions to N/D equations are not determined, even at low energies, by the contribution from exchange forces to the amplitude over the interval $4 < s < 68$ (in the notation of Refs. 1 and 2). Hence the information obtained about exchange forces in Ref. 2 was not sufficient to warrant the conclusion against ρ generation.

Having conceded that my earlier argument against ρ generation was inconclusive, I now wish to show that Dilley's argument in *favor* of ρ generation is also inconclusive. The demonstration proceeds by a counterexample, as follows:

The "successful" generation of a ρ by Dilley and co-workers[3] is based on the gap-matching method, wherein the distant left-hand cut used as input for N/D equations is varied until the output N/D agrees with the physical $A(s)$ over the "gap" $0 < s < 4$. This method is strongly biased in favor of generating a ρ, however, because the ρ corresponds to the largest nearby singularity in $A(s)$. Hence it would be difficult for any analytic function like N/D to match $A(s)$ over the gap, unless N/D had the *same* large nearby singularity, i.e., an output ρ resembling the physical ρ.

To demonstrate the aforementioned bias, we consider the following approximation for a resonance-dominated $A(s)$:

$$A(s) = \frac{s-4}{\pi}\left[\int_{-\infty}^{0} + \int_{4}^{\infty}\right] ds' \frac{\mathrm{Im}A(s')}{(s'-4)(s'-s)} \tag{1a}$$

$$\cong m_\rho \Gamma_\rho \frac{s-4}{(s_\rho - 4)(s_\rho - s)}, \tag{1b}$$

with $m_\rho = 5.07$ (i.e., 770 MeV) and $\Gamma_\rho = 1.09$ (i.e., 150 MeV). The amplitude (1b) is actually a rather good approximation to the physical $A(s)$ within the gap, being about six times larger than the (here neglected) contribution from the physical left-hand cut.[2] Unitarization of the right-hand cut would have little effect within the gap, since $A(s)$ vanishes at threshold, and the ρ is narrow relative to its mass.

Suppose now that we use a one-pole approximation for the left-hand cut of N, and vary the pole position and residue until agreement is maximized between N/D and the $A(s)$ of Eq. (1b), for $0 < s < 4$. If we interpret "maximum agreement" in terms of minimizing the integral

$$\Delta^2 \equiv \frac{1}{4} \int_0^4 ds \left(\frac{N/D - A}{A}\right)^2,$$

then the pole in N is uniquely determined,[4] and $\Delta_{\min}^2$ has the quite satisfactory value $\Delta_{\min}^2 = 8.2 \times 10^{-6}$. The resulting N/D has an *excellent* output ρ, with $m_\rho = 768$ MeV and $\Gamma_\rho = 155$ MeV. The phase shift even reaches a maximum value of 163° near 2 GeV, before beginning its slow descent back through 90° down to zero at $s = \infty$. The calculation is highly successful in producing a ρ in agreement with experiment, but *this cannot be regarded as evidence that the ρ in Eq. (1b) is generated by exchange forces*, because *the amplitude (1b) has no left-hand cut*. The "success" of this N/D calculation is merely evidence that the gap-matching method is strongly biased in favor of generating a ρ, *regardless* of whether the ρ in the $A(s)$ being matched is generated by forces in the $\pi\pi$ channel.

In conclusion, it appears that the calculations by myself and by Dilley and co-workers provide little if any evidence concerning the dynamical origin of the ρ. Numerous successes of the quark model suggest that low-lying resonances like the ρ are primarily diquark systems rather than dimeson systems, but this latter evidence is indirect, and is based in a different formalism.[5]

[1] J. Dilley, preceding paper, Phys. Rev. D 14, 2422 (1976).

[2] E. P. Tryon, Phys. Rev. D 12, 759 (1975).

[3] Cf. J. Dilley and R. Gibson, Nucl. Phys. B76, 69 (1974), and references cited therein.

[4] The left-hand cut of N is given by $\mathrm{Im}\,N = a\delta(s - \bar{s})$, with $\bar{s} = -2.848 \times 10^7$, and $a = 2.114 \times 10^{13}$. The resulting N/D agrees with the $A(s)$ of Eq. (1b) within 0.6% for $0 < s < 4$. One finds that $D(\bar{s}) = 7.523 \times 10^4$, so the pole in N/D corresponds to a left-hand cut with $\mathrm{Im}(N/D) = b\delta(s - \bar{s})$, where $b = 2.810 \times 10^8$. Since N/D satisfies a dispersion relation identical in form to Eq. (1a), the contribution to N/D from its left-hand cut is given by

$$(N/D)_L = \frac{s-4}{\pi} \frac{b}{(\bar{s}-4)(\bar{s}-s)}\,.$$

For $|s| \lesssim 10^5$, $(N/D)_L \cong 1.1 \times 10^{-7}(s-4)$, which is utterly negligible in the gap and low-energy region. In the limit as $s \to \infty$, however, $(N/D)_L$ tends to an asymptotic value of 3.14, which would imply a violation of unitarity were it not for the output resonance and slow return of the phase shift back down through 90° toward zero. The contribution to N/D from the resulting right-hand cut is large and *negative* in the asymptotic region (tending to -3.14 as $s \to \infty$), in such a way as to maintain unitarity.

[5] Cf. J. S. Kang and H. J. Schnitzer, Phys. Rev. D 12, 841 (1975); E. P. Tryon, Phys. Rev. Lett. 36, 455 (1976).